IDEA League - Homework

28. März 2015

1. **Energy levels of a Cooper-pair box**
   The Hamiltonian for a Cooper-pair box is given by

$$H_{CPB} = \sum_n \left[E_C(\hat{n} - n_g)^2 |n\rangle\langle n| - \frac{E_J}{2} \left(|n\rangle\langle n+1| + |n+1\rangle\langle n| \right) \right].$$

   (a) Write down the Hamiltonian for the two-dimensional qubit subspace in terms of the Pauli matrices σ_x and σ_z by restricting the quantum states to $n = 0, 1$. **Hint:** Set your reference energy between the two eigenstates.

   (b) What is the transition frequency between ground and excited state as a function of n_g?

   (c) Plot the energy levels of H_{CPB} for $E_J/E_C = 2$ as a function of n_g. Truncate your matrix after the lowest 10 charge states.

   (d) For $E_J/E_C = 2$ and $n_g = 5.5$, what are the transitions frequencies between the lowest 3 energy bands.

   (e) Describe what happens to the dependence of the energy levels on n_g for $E_j \to \infty$. In this limit, what are now the transitions frequencies between the lowest 3 energy bands. What is the energy difference between the lowest and highest energy band.

2. **Resonator-qubit interaction in dispersive regime**
The coupling of a qubit to a resonator in the rotating wave approximation is written as

$$H = \hbar\omega_r \left(a^\dagger a + \frac{1}{2} \right) - \frac{\hbar\omega_q}{2}\sigma_z + \hbar g(a^\dagger\sigma^- + a\sigma^+).$$

Here, ω_r and ω_q are the frequencies of the resonator and the qubit, respectively, and g is their coupling strength.

In the dispersive regime when the qubit and the resonator frequencies are far detuned ($|\delta| = |\omega_q - \omega_r| \gg g$), diagonalization of this Hamiltonian to the lowest order in g leads to

$$H = \left(\hbar\omega_r + \frac{\hbar g^2}{\Delta}\sigma_z \right) \left(a^\dagger a + \frac{1}{2} \right) - \frac{\hbar\omega_q}{2}\sigma_z.$$

(a) If we can only measure transmission and reflection of the resonator, how can we infer the state of the qubit? Does the amplitude of the drive (or equivalently the number of photons in the resonator) lead to problems? If yes, how to we need to choose it?

(b) How does the qubit energy depend on the resonator state?

(c) Modulating the gate charge n_g, see Exercise 1, with a pulse at a frequency ω_g we can change the state of the qubit. Using the transmission through the resonator, how can we infer the state of the resonator using the qubit? Again, does the amplitude of the drive lead to problems? If yes, how do we need to choose it?