IDEA League QIP School RWTH Aachen

Exercises: Intro/Superconducting Qubit Lectures by D. P. DiVincenzo

To be returned no later than March 20, 2015 to local coordinator

1. Unextendable Product Bases for two qubits?

a. In the lectures we saw that in two-qutrit systems, one finds that in the process of enumerating orthogonal product states, we can get complete bases that are unmeasurable, and we can also encounter "stoppers", so that we end up with a UPBs (always at the 5th state, it turns out). Your assignment is to construct an argument that this never happens for two qubits (2x2 Hilbert space). Build up a general product basis, one state at a time, and observe that you cannot get stuck. Your result should be the most general complete product basis (four orthogonal quantum states). Hint: we will not distinguish bases that are related by local qubit rotations; therefore, you can without loss of generality start with the state $|00\rangle$.

b. (optional) If this was child's play for you, work out the same result for the case where Alice has a qubit and Bob has an n-level system, for any n. There is also no UPB for 2xn for any n. You can read about this in

http://arxiv.org/abs/quant-ph/9808030v1

2. Partial projective measurement for qutrits.

Transmon qubits provide a realisation of 3-level quantum systems. When coupled to a cavity mode of frequency ω_r, there is a regime (within the Jaynes-Cummings picture), the so-called dispersive regime, in which, if the qubit is in the state $|i\rangle$, $i = 0, 1, 2$, then the frequency of the cavity mode is shifted to $\omega_r + \chi_i$, with χ_i given by (derived in J. Gambetta, Juelich IFF Spring School Lecture Notes, 2012, and S. Richer, Masters Thesis, RWTH-IQI, 2013):

$$\chi_i = g^2(\mu_{i+1} - \mu_i), \;\; \mu_i = \frac{i}{\omega_r - \omega_{01} - (i-1)\delta} \tag{0.1}$$

ω_{01} is the 0-1 splitting of the transmon; the 1-2 splitting is shifted by δ. By measuring the reflection of a microwave tone from the cavity, the resonant frequency can be accurately measured, and as long as there is a different frequency for each qubit state, a complete von Neumann measurement is accomplished.

The assignment is this: I stated in the lecture that it is valuable sometimes to have only a two-outcome measurement, where 0 is distinguished from 1 and 2 , but 1 and 2 are *not* distinguished. Please "design" this measurement. Consider ω_{01} as a tuneable parameter. Redo the design with the objective to achieve a two outcome measurement in which 0 and 2 are not distinguished.