IDEA League QIP School RWTH Aachen

Exercises: Intro/Superconducting Qubit Lectures by D. P. DiVincenzo

To be returned no later than March 20, 2015 to local coordinator

1. Unextendable Product Bases for two qubits?

a. In the lectures we saw that in two-qutrit systems, one finds that in the process of enumerating orthogonal product states, we can get complete bases that are unmeasurable, and we can also encounter "stoppers", so that we end up with a UPBs (always at the 5th state, it turns out). Your assignment is to construct an argument that this never happens for two qubits (2x2 Hilbert space). Build up a general product basis, one state at a time, and observe that you cannot get stuck. Your result should be the most general complete product basis (four orthogonal quantum states). Hint: we will not distinguish bases that are related by local qubit rotations; therefore, you can without loss of generality start with the state $|00\rangle$.

b. (optional) If this was child's play for you, work out the same result for the case where Alice has a qubit and Bob has an n-level system, for any n. There is also no UPB for 2xn for any n. You can read about this in

http://arxiv.org/abs/quant-ph/9808030v1

2. Partial projective measurement for qutrits.

Transmon qubits provide a realisation of 3-level quantum systems. When coupled to a cavity mode of frequency ω_r, there is a regime (within the Jaynes-Cummings picture), the so-called dispersive regime, in which, if the qubit is in the state $|i\rangle$, $i = 0, 1, 2$, then the frequency of the cavity mode is shifted to $\omega_r + \chi_i$, with χ_i given by (derived in J. Gambetta, Juelich IFF Spring School Lecture Notes, 2012, and S. Richer, Masters Thesis, RWTH-IQI, 2013):

$$\chi_i = g^2(\mu_{i+1} - \mu_i), \;\; \mu_i = \frac{i}{\omega_r - \omega_{01} - (i-1)\delta} \tag{0.1}$$

ω_{01} is the 0-1 splitting of the transmon; the 1-2 splitting is shifted by δ. By measuring the reflection of a microwave tone from the cavity, the resonant frequency can be accurately measured, and as long as there is a different frequency for each qubit state, a complete von Neumann measurement is accomplished.

The assignment is this: I stated in the lecture that it is valuable sometimes to have only a two-outcome measurement, where 0 is distinguished from 1 and 2 , but 1 and 2 are *not* distinguished. Please "design" this measurement. Consider ω_{01} as a tuneable parameter. Redo the design with the objective to achieve a two outcome measurement in which 0 and 2 are not distinguished.

Answers.

1. Unextendable Product Bases for two qubits?

a. There is no UPBs for 2 qubits. We start enumerating a product basis and see that it is always completable. As stated in the problem statement, we start with $|00\rangle$. Without loss of generality, we can take the second product state to be orthogonal to the first on the A side. Thus, the second state has the form $|1\rangle_A \otimes (\alpha|0\rangle + \beta|1\rangle)_B$, for arbitrary α and β. If α and β are nonzero, then the third state must be be orthogonal on the A side for one state and the B side for the other state; there is not enough room in two dimensions to be orthogonal to two other different vectors. The case of α or $\beta = 0$ works out obviously, we then have two members of the standard basis. Dealing with the generic case, we see that there is a third state, namely $|01\rangle$. The fourth state of the basis is now unique, and it is a product state: $|1\rangle_A \otimes (\beta^*|0\rangle - \alpha^*|1\rangle)_B$. If one had chosen the opposite orthogonality pattern, one would have arrived at the same final two states, but in the opposite order.

b. As mentioned in the problem statement, the 2xn case is dealt with in
http://arxiv.org/abs/quant-ph/9808030v1

Please note that this is "v1" of this paper, the argument was removed in v2 and in the published paper. The idea is to relate computability of LOCC measurability.

2. Partial projective measurement for qutrits.

In this scheme we measure a dispersive shift given by

$$\chi_i = g^2 \frac{i}{\omega_r - \omega_{01} - (i-1)\delta} \tag{0.2}$$

Which, for the first three states of the transmon, is explicitly

$$\begin{aligned}
\chi_0 &= \frac{g^2}{\omega_r - \omega_{01}} \\
\chi_1 &= \frac{2g^2}{\omega_r - \omega_{01} - \delta} - \frac{g^2}{\omega_r - \omega_{01}} \\
\chi_2 &= \frac{3g^2}{\omega_r - \omega_{01} - 2\delta} - \frac{2g^2}{\omega_r - \omega_{01} - \delta}
\end{aligned}$$

The states 1 and 2 are indistinguishable in the measurement if $\chi_1 = \chi_2$, and this happens when $\omega_{01} = \omega_r + \delta$. The states 0 and 2 are indistinguishable in the measurement if $\chi_0 = \chi_2$, and this happens when $\omega_{01} = \omega_r - \delta/2$. It can be confirmed that in both cases the third χ_i is distinct – thus, the other state is indeed distinguished by the frequency measurement.