IDEA League QIP School RWTH Aachen

Exercises Topological Quantum Computing Lectures by Fabian Hassler

To be returned no later than March 20, 2015 to local coordinator

Hint: For this exercise you should remember that the Majorana operators are Hermitian and obey the Clifford algebra $\{\gamma_k, \gamma_l\} = 2\delta_{kl}$.

1 Fusion three Majorana fermions

The fusion of three Majorana fermions $\gamma_1, \gamma_2, \gamma_3$ is described by the Hamiltonian

$$H = \sum_{kl} \Delta_{kl} \gamma_k \gamma_l.$$

(a) Split the general matrix $\Delta = \Delta^s + \Delta^a$ into the symmetric $\Delta^s = \frac{1}{2}(\Delta + \Delta^t)$ and the antisymmetric $\Delta^a = \frac{1}{2}(\Delta - \Delta^t)$ part. Are both parts relevant in the expression for the Hamiltonian H?

(b) What are the properties that the matrix Δ needs to have in order that H constitutes a Hermitian operator?

(c) From (a) and (b) you should have learned that Δ is in general an imaginary antisymmetric 3×3 matrix. Write down the most general antisymmetric 3×3 matrix and determine its eigenvalues.

(d) Prove in general that any antisymmetric imaginary $n \times n$ matrix with n odd has a zero eigenvalues. Physically this means that fusing an odd number of Majorana fermions there will be always a Majorana fermion (=zero mode) left.

2 Majorana representation of the braid group

We have seen in the lecture that exchanging Majorana γ_k with γ_{k+1} in a counterclockwise fashion is implemented by the operator

$$B_k = \exp\left(\frac{\pi}{4}\gamma_k \gamma_{k+1}\right).$$

(a) Convince yourselft that B_k is a unitary operator.

(b) What is the expersion for B_k^{-1}, i.e., the clockwise exchange of the two particles.

(c) Show that B_k constitute a representation of the braid group. For that you have to show that it fulfills the two defining properties of the braid group

$$[B_k, B_l] = 0, \quad \forall |k-l| \geq 2 \qquad \text{and} \qquad B_k B_{k+1} B_k = B_{k+1} B_k B_{k+1}, \quad \forall k.$$