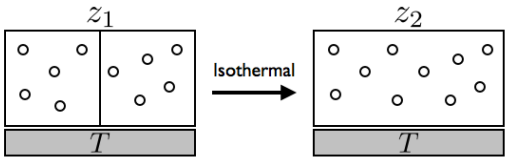
IDEA League QIP School RWTH Aachen

Exercises on the Quantum Thermodynamics Lectures by Renato Renner

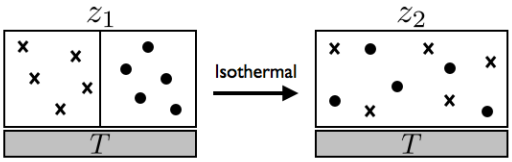
To be returned no later than March 20, 2015 to local coordinator

1. Gibbs paradox

(a) Consider a container consisting of two compartments separated by a removable wall. The two compartments are filled with gas at the same pressure and temperature. Assume first that you are not aware of any process that can distinguish the gas in the left compartment from that in the right one. By how much does the entropy of the container change when you remove the wall?

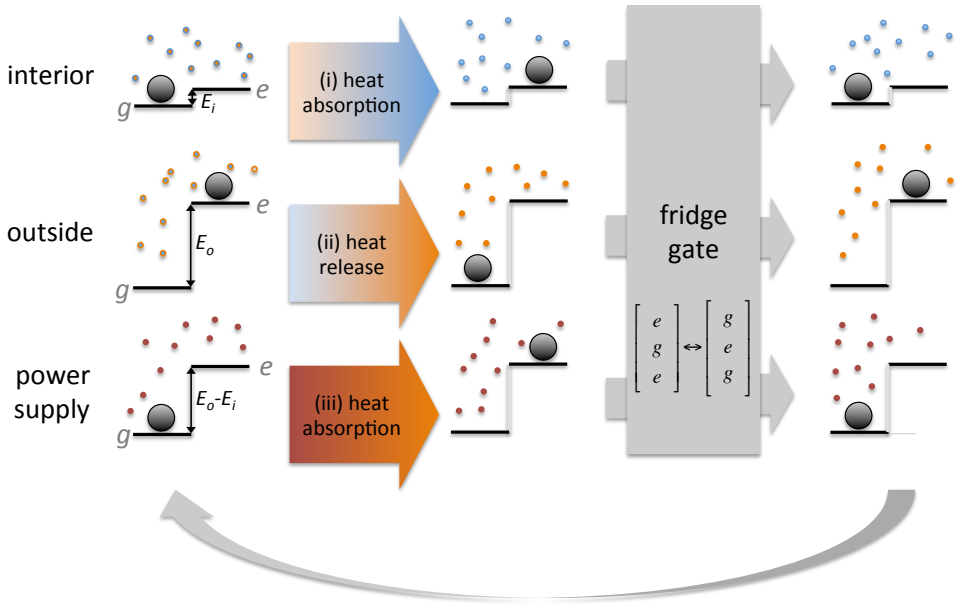


(b) Assume now that you discover two materials, A and B, and that you find that the gas in the left compartment can pass through A but not through B, and that the gas in the right compartment can pass through B but not through A. This shows, in particular, that the two compartments are actually filled with two different types of gas. By how much does the entropy now change when you remove the wall? Carry out this calculation both in the framework of Phenomenological Thermodynamics and in Statistical Mechanics.



2. Qubit-based thermodynamic machines

It is possible to build a cooling device (a "fridge") that consists of just three qubits: one of them is coupled to a cold reservoir at temperature T_i (the "inside" of the fridge), one to a hot reservoir at temperature T_o (the "outside" of the fridge), and one to a power supply.[1] The mechanism uses work from the power supply to transfer heat from the cold to the hot reservoir. Specifically, cooling is achieved by repeated execution of the "fridge gate" depicted in the figure. Furthermore, it is assumed that after each execution of the gate, the qubits coupled to the heat reservoirs thermalise so that their density operators are diagonal in the energy eigenbasis and their eigenvalues are given by the Gibbs distribution.



(a) Depending on the energy gaps E_i and E_o as well as the temperatures T_i and T_o, determine the expected amount of heat transferred to the two reservoirs during one execution of the gate.

(b) How does one need to choose E_i and E_o so that the process is maximally efficient?

[1] The mechanism was proposed by Skrzypczyk *et al.* in *J. Phys.* A **44**, 492002 (2011); see also *Nature* **482**, pp. 164–165 (2012).