Problem sets for IDEA League QI School
Control and coherence of semiconductor spin qubits
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These problems do not require complicated calculations, but a good understanding of the underlying physics. If required information is missing make reasonable assumptions or refer to the literature.

Exercise 1: Quantum dot energy scales (8 Points)

In physics is it often important to get a sense for the energy scales involved in a problem. In the lecture you learned about the quantum dot potential used to trap electrons. In this exercise you will estimate the characteristic energies associated with GaAs quantum dots. Express all energy scales in eV and compare them to the thermal energy at $T = 100$ mK.

Note: The effective electron (band) mass in GaAs is $m^* = 0.07m_e$ (m_e is the free electron mass). The dielectric constant of GaAs $\epsilon_r = 13.1$.

1. The conduction band bottom of the most common type of 2DEG in GaAs/AlGaAs heterostructure has a triangular shape as a function of the distance from the sample surface, which confines electrons in this direction. The corresponding eigenenergies and -states can be calculated numerically. Here, we assume for simplicity instead that the 2DEG is hosted in a square quantum well with a thickness of $d_z = 10\,\text{nm}$. Calculate the energy difference between the lowest and second lowest eigenstate of the z-degree of freedom.

2. Now we turn on negative voltages on some of the metallic gates, to form a single dot with a parabolic potential in the xy-plane (i.e., the plane of the 2DEG):

$$V(x, y) = \frac{1}{2}k(x^2 + y^2),$$

   where the value of the spring constant k is such that the potential rises by 2 meV at 50 nm from its minimum (this is typical). Calculate the energy required to excite the electron to the next orbital in this confinement potential. Sketch the shape of the electron wavefunction of the ground state and give an estimate of its width Δx. (Hint: use known results from the harmonic oscillator.)

3. What is the charging energy required to charge the dot with a second electron? To calculate this Coulomb energy assume the dot to be a homogenously charged sphere with a radius corresponding to $\Delta x/2$ from part 2 (use 20 nm if you did not solve part 2) and estimate the energy needed to bring a second electron to the surface of the sphere.

4. A second dot is formed at a distance $d = 180\,\text{nm}$ from the first dot. Estimate the inter-dot coupling energy, i.e., how much the Coulomb energy required to fill the second dot changes depending on whether an electron is present in the first dot.

5. Sketch a charge stability diagram of the double dot (dot occupation as a function of voltages on left and right gate as discussed in the lecture) and indicate which feature is most directly related to the interdot coupling energy.

Exercise 2: Energy diagram of the charge qubit (8 Points)

1. A double quantum dot occupied by a single electron can be operated as a charge qubit. Sketch its energy diagram (energies of eigenstates as a function of detuning ϵ).
2. Write down the two-level Hamiltonian of the charge qubit in the basis $|L\rangle$, $|R\rangle$ with the electron being entirely on the left or right.
3. The eigenstates $|\psi_i\rangle$ ($i = \{1, 2\}$) are a function of the detuning as well. Assuming the Hamiltonian from part 2, show that the local slope $\frac{dE_i}{d\epsilon}$ (at a given ϵ) indicates how much $|L\rangle$ component the eigenstates $|\psi_i\rangle$ contains. This relation tells you that the energy diagram also reveals the nature of the eigenstates. Hint: perturbation theory can simplify the job.
4. Consider a single-electron charge qubit that is controlled by electrostatic gate electrodes that are connected by a parallel circuit of a 1 pF capacitor and a 1 MΩ resistor. Assume that the potential difference between the two sites of the double dot is 1/10 of the voltage difference between the gate electrodes and that the resistor has a temperature of 100 mK. Determine (approximately) the form of the decay of the qubit coherence and the associated coherence time. Is it T_2 or T_2^*? Hint: first compute the variance and correlation time of the voltage.

Exercise 3: Filter function of the free induction decay (6 Points)

In a free induction decay (FID) experiment, a qubit is initialized in an equal superposition of the two eigenstates, left to evolve for some time τ, and then measured in the basis containing the initial state.

Derive the relation discussed in the lecture between the dephasing factor $\exp(-\langle\delta\phi^2\rangle/2)$ and the spectral density of a noise process seen by the qubit as a function of τ, where $\langle\delta\phi^2\rangle$ is the variance of the qubit phase uncertainty resulting from the noise. Note: the spectral density of a (stationary) noise process $\beta(t)$ is given by

$$S_\beta(\omega) = \int dt\, e^{i\omega t}\langle\beta(t')\beta(t'+t)\rangle,$$

which by definition is independent of t'. Hint: Write the dephasing factor in terms of the correlator $\langle\beta(t')\beta(t'+t)$ and then use Fourier transforms.